A Contribution to the Modeling of Inductors Printed on Flexible Substrates

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Abstract— **The purpose of in-house developed software tool is calculating the inductances of conductors printed on flexible substrates. The inductances are obtained by integration through the conductors. In this paper, a method for fine modeling of the edges of conductors is explained. Current density is obtained from boundary conditions on the conductor's edges. The results are validated by finite element simulations.**

Keywords- inductors ; printed electronics; modeling

I. INTRODUCTION

Recent developments in printed and flexible electronics have provided numerous new applications [1]. Design of flexible components can be helped by computer programs aimed at predicting their performances. In this paper, we study an improvement for a computer program written in C++ designed for accurate modeling of inductors printed on flexible substrate. This program has already been tested by comparison with experimental results [2] and analytical expressions [3] and very good agreement has been found. However, in the case of meander inductors, the model can be improved by adding the edge effects. In the following paragraph the basic model is presented and some representative results are given. In the next paragraph, the improved model is described and potential and current density obtained by this model are validated through comparison with finite element method. Finally, a conclusion with intended future work is given.

II. INDUCTANCE CALCULATION

The inductors printed on flexible substrates are a specific electromagnetic problem: printed conductors are very thin and usually there are no ferromagnetic materials in the domain of study. These facts make the inductance calculation of such conductors by finite elements time and memory consuming and it is difficult to achieve high accuracy. This is due to the meshing problems (caused by thin conductors and the need for meshing the empty space) and domain truncation error. For these reasons, a software package called *Provod* has been developed, based on calculation of vector potential by integration of current density through the conductors. In this way, only the conductors are divided into elements – there is no need for meshing the empty space. By this method, the infinite domain is always taken into account. In addition, this method allows fast computation by using parallel computing

[4]. Finally, an in-house developed program allows us to simplify entering of geometry data by using pre-programmed curved shapes of the flexible substrate.

A. Basic inductance calculation method

All conductors in the domain of study are divided into small volume elements. The inductance is calculated by integrating over the elements using the following expression [5]:

$$L = \frac{1}{I^2} \int \vec{J}\vec{A}dv \qquad (1)$$

where: J – current density vector in the element; A – vector potential at the center of the element; dv – volume of the element. The vector potential at a point is computed by integration through all the other elements in the problem [5]:

$$\vec{A} = \frac{\mu}{4\pi} \int \frac{\vec{J}}{R} dv \qquad (2)$$

where: R – distance between elements; μ – magnetic permeability (of vacuum, in our case). The integrals in (1) and (2) are calculated as sums over all the elements in all conductors, excluding the same element (R=0). In Figure 1, an overview of the method is shown. The vector potential of the element 1 is obtained by taking into account current density and distance of all the other elements with respect to the center of the element 1. For example, in Figure 1 the element 2, its current density and its distance R from the element 1 are drawn.
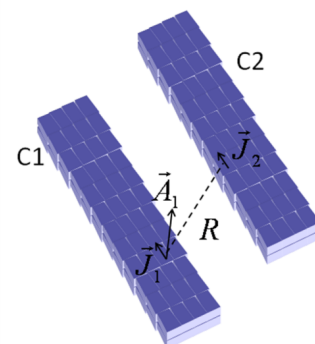

Figure 1.   Elements on conductors C1 and C2

The initial geometry of printed components is defined by the printed pattern in a plane. In the *Provod*, this pattern is entered as set of straight segments in u-v plane connected in series (please see example in Figure 2), each segment having uniform current density.
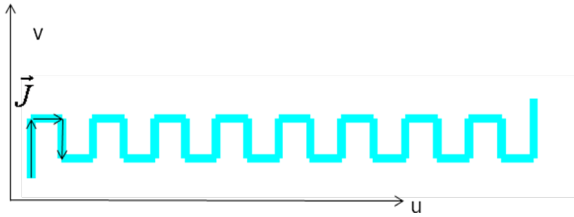

Figure 2.   Segments of a conductor in u-v plane

The deformation of a flexible substrate is taken into account when three-dimensional coordinates of each element are calculated [4]. The u-v coordinates of elements obtained by the initial geometry input are transformed by the program into x-y-z coordinate system. The deformation of flexible substrate is defined by using appropriate mathematical expressions. In this manner, the user does not need to draw complicated geometries in three dimensions and the structure is described in the same way as it is fabricated afterwards – first it is printed on a flat support and then the substrate may be deformed.

The accuracy of the inductance calculation in *Provod* has already been confirmed in previously published papers [2], [3], [4], [6]. The accuracy has been tested for flat and curved substrates. Curved substrates have been tested for two cases: spiral and circular (and other) shapes, due to the separate representation of spirals in the code.

III.   CURRENT DENSITY CALCULATION

The use of uniform current density for inductance calculation, as it is described in section II, is an approximation which is valid in many practical cases, but the goal of presented work is to make the model closer to the reality. Computing the current distribution for general case of any conductor shape would be demanding and would change the whole software structure. However, it is possible to include a simple model of a corner in meander inductors, where the current density now depends on 2D coordinates of an element. In this section, the calculation of current density from boundary conditions is described, and the obtained values are compared with finite element method.

A.   Proposed modification of meander representation

A meander consists of straight-line segments. If we consider that current density is uniform in the middle of a segment, we should consider corners as the parts of meander where the current density (intensity and direction) depends on coordinates.

B.   Current density calculation method

The current density in any conductor is given by the formula [7]:

$$\vec{J} = \sigma\vec{E} \tag{3}$$

where σ represents the conductivity and E the electric field.

If we assume σ = 1, the current density is obtained as negative gradient of electric scalar potential inside a conductor:

$$\vec{J} = -\nabla V \tag{4}$$

From Maxwell's equations for the case of DC conductors, Laplace's equation for the potential is obtained [7]:

$$\nabla^2 V = 0 \tag{5}$$

For functions which satisfy the equation (5), the value of potential at a point inside a region can be expressed as an integral of boundary values of the potential and its derivative on the region boundary Γ, as follows [7]:

$$V = \int w \frac{\partial V}{\partial n} d\Gamma - \int \frac{\partial w}{\partial n} V d\Gamma \tag{6}$$

where w is the potential function in two dimensions, equal to $-(1/2\pi)\log(r)$, r being the distance from the point of integration to the point at which the potential is calculated.

We can now apply (6) to the case of corner which belongs to a conducting meander. An example is shown in Figure 3. First finite element simulation is made in order to determine the corner size, we find that if side 1 is 2mm long, side 2 should be 5 mm long. These dimensions have been studied in the presented examples.
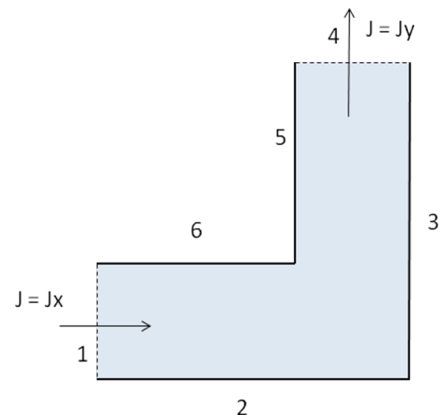

Figure 3.   Corner of a conductor in a plane

For the sides 2, 3, 5 and 6 the following boundary condition is valid: $\frac{\partial V}{\partial n} = 0$.

For the sides 1 and 4 the boundary conditions are $V=V_0$ and $V=0$, respectively. The value of V_0 is an arbitrary choice,

which can be used for scaling the obtained solution. The condition imposed in *Provod* software is that total current should be equal to 1. V_0 used in further text is obtained from finite element simulation in two dimensions with total current equal to 1 (model height is supposed to be 1 meter) and its value is 3.56 V for the given dimensions.

From (4), the gradient of (6) will give us the current density as a function of boundary conditions:

$$\vec{J} = -\int (\nabla w)\frac{\partial V}{\partial n}d\Gamma + \int \nabla(\frac{\partial w}{\partial n})Vd\Gamma \qquad (7)$$

The expression (7) has been applied in the *Provod* software tool.

C.   Results and validation

The potential and current density have been calculated by (6) and (7) and by finite elements on three different paths A, B and C inside a conductive corner (please see Figure 4). These paths are equally spaced with respect to the conductor edges (at 0.5 mm distance).
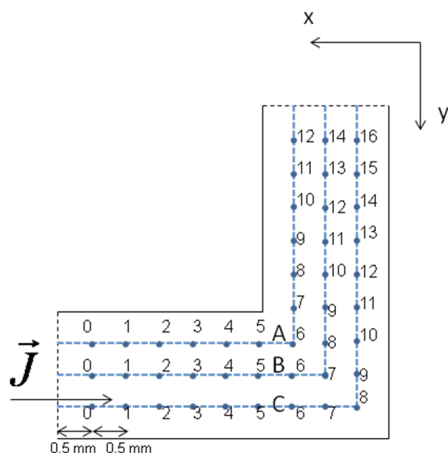

Figure 4.   Paths A, B and C on a current conducting corner

The potential on these three paths, for points with 0.5 mm spacing, starting at 0.5 mm from the end of each path (Figure 4), is given in Figures 5, 6 and 7, point numbers following the direction of the current.
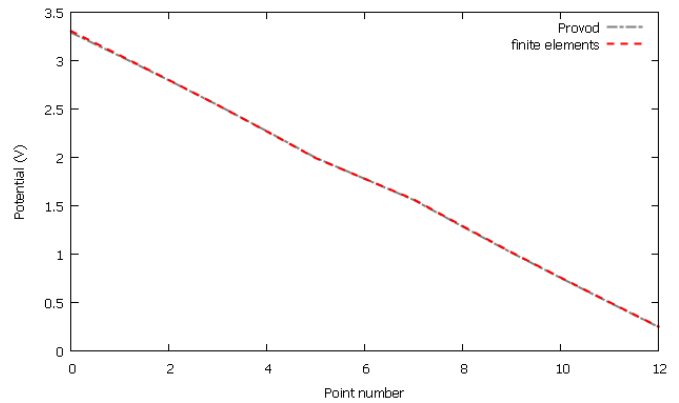

Figure 5.   Potential on path A from *Provod* and finite elements software
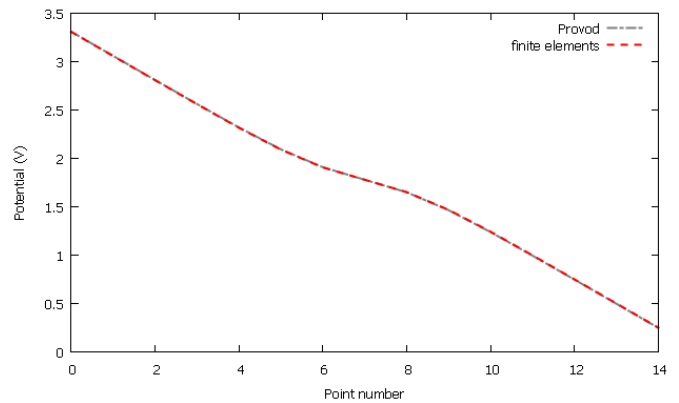

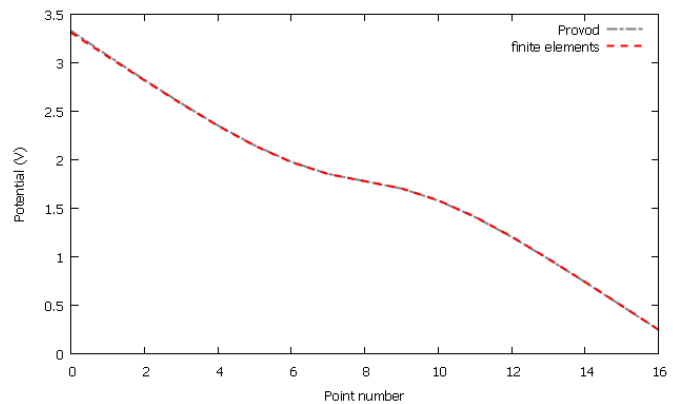Figure 6.   Potential on path B from *Provod* and finite elements software

Figure 7.   Potential on path C from *Provod* and finite elements software

The potentials computed in the *Provod* on all three paths are in excellent agreement with the potentials obtained by the finite element method. The potential varies from a value close to 3.56 V (V_0, at one end) to the value close to 0 (at the other end). We observe that the variation of potential at the middle points (at the corner) is the greatest for the path A (the most inner path) and it is the smallest for the path C which is the longest path.

In Figures 8, 9 and 10 the partial derivative of potential with respect to x axis on paths A, B, and C (shown in Figure 4) is given.
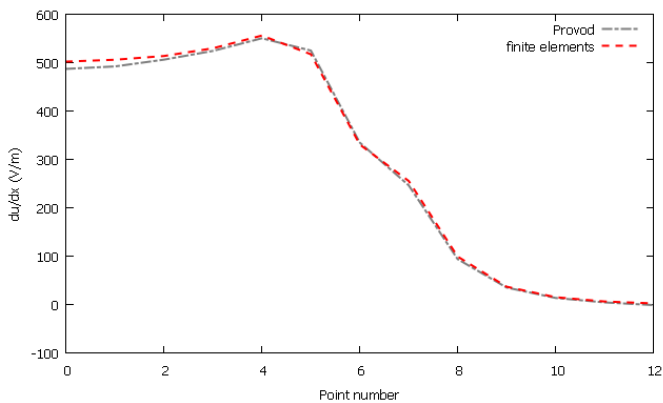

Figure 8.   Derivative of potential with respect to x on path A from *Provod* and finite elements software
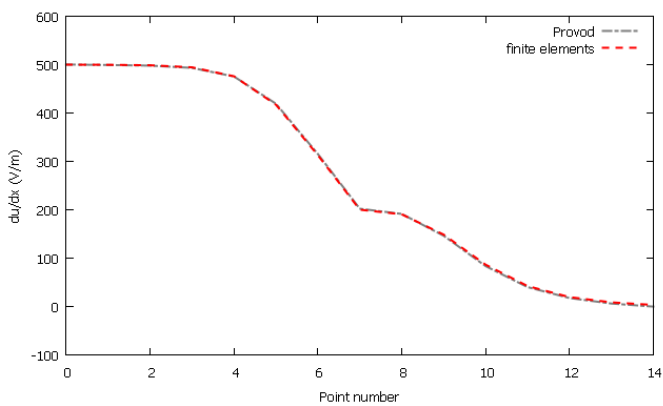

Figure 9.   Derivative of potential with respect to x on path B from *Provod* and finite elements software
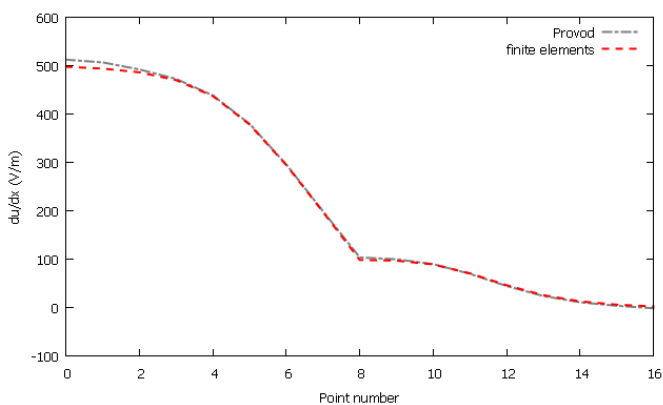

Figure 10.   Derivative of potential with respect to x on path C from *Provod* and finite elements

The derivatives of potential with respect to x match very closely the values obtained from finite elements. We expect the current density to have only x component at the end where the conductor lies on x axis and, at this end, all the current flows on x axis. The value of current density is 500, this is the consequence of adjusting V_0 in order to have total current (current density multiplied by the cross-section surface, which

is equal to 0.002 square meters) equal to 1. In *Provod* software, the current density should be further scaled to take into account the conductor thickness. At the other end, where the conductor lies on the y axis, the current should flow on y axis and the x component of the current density should be close to 0. This behavior is observed in Figures 8, 9 and 10. Each path has a corner point in the middle which explains the discontinuity of the slope in the middle of each curve.

In Figures 11, 12 and 13 the partial derivative of potential with respect to y axis on paths A,B and C (shown in Figure 4) is represented.
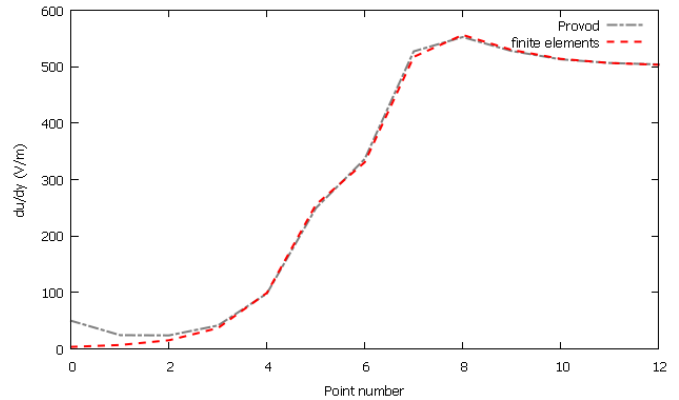

Figure 11. Derivative of potential with respect to y on path A from *Provod* and finite elements software
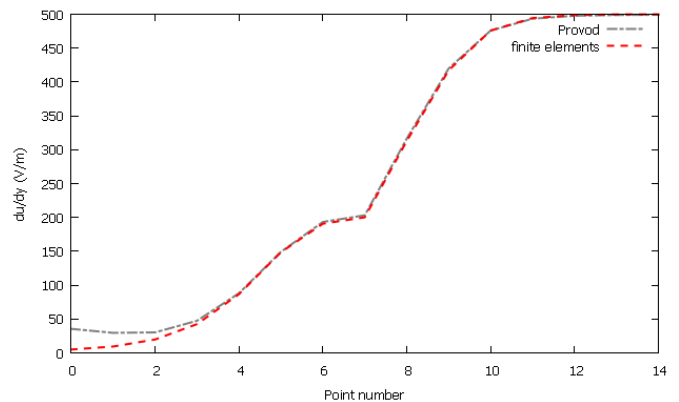

Figure 12. Derivative of potential with respect to y on path B from *Provod* and finite elements software
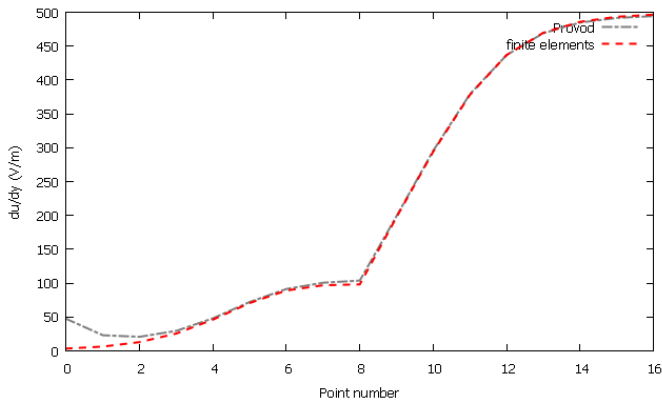
Figure 13. Derivative of potential with respect to y on path C from *Provod* and finite elements software

The derivatives of potential with respect to y match closely the values obtained from finite elements, the only small discrepancies are observed for the smallest values of derivative at first two points. As expected, there is almost no y component of the current at the end where the conductor is placed on x axis. At the other end, all current flows on y axis. The discontinuities of slopes are present at corner points, as expected.

General behavior of the current density is well observed, the change of direction from x to y is found – at one end only the x component is present and it vanishes at the other end and y component behaves accordingly. The values of current density match the finite element simulations.

IV. CONCLUSION

The inductance of flexible meander conductors can be calculated by taking into account the edge effects. Current density values obtained from the boundary conditions for a corner have been validated by comparison with finite element results. The conductors studied in this paper have corners with 90° angle, but the presented method can be applied for any angle. Further work will consist in implementing this calculation into the code for the inductance calculation, in order to improve the accuracy of inductance calculation of printed conductors.

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